Enrollment No. ____________

Shree Manibhai Virani and Smt. Navalben Virani Science College (Autonomous), Rajkot

Affiliated to Saurashtra University, Rajkot

SEMESTER END EXAMINATION NOVEMBER - 2017

M.Sc. Mathematics

16PMTCC12 – COMPLEX ANALYSIS

Duration of Exam – 3 hrs ***Semester – III*** ***Max. Marks – 70***

***Part A** (5X2= 10 marks)*

Answer **ALL** questions

1. Which subset of the unit sphere S corresponds to the imaginary axes in $\mathbb{C}$?
2. Define branch of the logarithm and comment about its derivative.
3. Define line integral of f along $\gamma:[a,b] \to \mathbb{C}$.
4. If $f(z) = \frac{(1+z)^n}{z^{k+1}}$ then find $\operatorname{Res}(f(z),0)$
5. Explain Laurent's series representation for an analytic function.

***Part B** (5X5= 25 marks)*

Answer **ALL** questions

6a. Show that $f : G \to \mathbb{C}$ is differentiable at a point a in G then f is continuous at a

OR

6b. Find the cube root of $-8i$.

7a. Give an example of a path which is not rectifiable.

OR

7b. If $\gamma : [a,b] \to \mathbb{C}$ is piecewise smooth then prove that γ is of bounded variation and

$$V(\gamma) = \int_a^b |\gamma'(t)| dt$$

8a. Prove that

$$\int_0^\pi \frac{e^{is}}{e^{is} - z} ds = 2\pi, \forall z \in \mathbb{C}, |z| < 1$$

OR

8b. State and prove Cauchy's theorem for open disc.

9a. State and prove open mapping theorem.

OR

9b. State and prove minimum modulus theorem.

10a. State and prove argument principle.

OR

10b. If f has a pole of order n at z_0 then prove that $\operatorname{Re}s(f(z),z_0)=\dfrac{1}{(n-1)!}\dfrac{d^{n-1}}{dz^{n-1}}(z-z_0)^n f(z)$

<u>Part C</u> (5X7= 35 marks)

Answer **<u>ALL</u>** questions

11a. State and prove Lebnitz' Rule.

OR

11b. Let $f:G\to\mathbb{C}$ be analytic and suppose $\bar{B}(a,r)\subset G; r>0$. If $\gamma(t)=a+re^{it}; 0\le t\le 2\pi$ then $f(z)=\dfrac{1}{2\pi i}\displaystyle\int_\gamma \frac{f(w)}{w-z}dw$ for $|z-a|<r$.

12a. State and prove Morera's theorem.

OR

12b. Let G be an open subset of the plane and $f:G\to\mathbb{C}$ is analytic function. If $\gamma_1,\gamma_2,...,\gamma_m$ are closed rectifiable curves in G such that $n(\gamma_1,w)+n(\gamma_2,w)+...+n(\gamma_m,w)=0, \forall w\in\mathbb{C}-G$, then for $a\in G-\{\gamma\}$ and $k\ge 1$, $f^k(a)\displaystyle\sum_{j=1}^{m} n(\gamma_j;a)=k!\sum_{j=1}^{m}\frac{1}{2\pi i}\int_{\gamma_j}\frac{f(z)}{(z-a)^{k+1}}dz$.

13a. Explain Schwarz' lemma.

OR

13b. State and prove inverse function theorem.

14a. Show that $\displaystyle\int_0^\infty \frac{\cos ax}{(x^2+1)^2}dx=\frac{\pi}{4}(a+1)e^{-a}; a>0$

OR

14b. If f has an isolated singularity at a, then the point $z=a$ is a removable singularity if and only if $\lim\limits_{z\to a}(z-a)f(z)=0$.

15a. How many roots has the equation $z^4+z^3-4z+1=0$ in $1<|z|<3$?

OR

15b. Show that $\displaystyle\int_{-\infty}^{\infty}\frac{dx}{x^2+1}=\frac{\pi}{\sqrt{12}}$.